

Axial Anomaly in Quasi-1D Chiral Superfluids

J. Goryo

Department of Physics, Hokkaido University, Sapporo, 060-0810 Japan
(February 1, 2008)

The axial anomaly in a quasi-one-dimensional (quasi-1D) chiral p -wave superfluid model, which has a $\varepsilon_x p_x + i\varepsilon_y p_y$ -wave gap in 2D is studied. The anomaly causes an accumulation of the quasiparticle and a quantized chiral current density in an inhomogeneous magnetic field. These effects are related to the winding number of the gap. By varying the parameters ε_x and ε_y , the model could be applicable to Sr_2RuO_4 near the second superconducting transition point, some quasi-1D organic superconductors and the fractional quantum Hall state at $\nu = 5/2$ Landau level filling factor.

PACS numbers: 74.25.Ha, 03.70.+k, 73.40.Hm

The chiral superfluidity is realized in the superfluid $^3\text{He-A}$ [1]. Recently, the possibility of the chiral superconductivity is argued [2,3]. In such superfluids or superconductors, the ground state is the condensate of the Cooper pairs which have orbital angular momentum along a same direction. Therefore, time-reversal symmetry (T) and also parity (P) in two-dimensional space (2D) are violating. We investigate a quasi-1D chiral p -wave superfluid in 2D. It is revealed that the axial anomaly causes P- and T-violating phenomena related to the quantized number.

The axial anomaly has been originally pointed out in the Dirac QED in 3D [4,5]. It is a phenomenon that a symmetry under the phase transformation $e^{i\gamma_5\alpha}$ of the Dirac field in the action at the classical level is broken in the quantum theory. Here, α is a constant and γ_5 is a hermitian matrix which anti-commutes with all of the Dirac matrices γ_μ , where μ is the spacetime index. The Adler-Bardeen's theorem guarantees the absence of higher order collections to the divergence of the axial current [4]. Therefore, the exact calculation of the two-photon decay rate of neutral π meson can be done. It has been pointed out that the same results are obtained by using the path-integral formalism and has been clarified the relation between the axial anomaly and topological quantized numbers through the Atiyah-Singer index theorem [6].

It has been pointed out that the axial anomaly also plays important role in the quantum Hall effect (QHE) in the 2D massive Dirac QED. In 2D, the mass term of the Dirac Fermion violates P and T like the magnetic field, and the Hall effect may occur. It was shown that the existence of the Hall current and its quantization are caused by the axial anomaly in 1D [7,8]. The relation between the axial anomaly and QHE in 2D electron gas in the magnetic field was also discussed [8,9], and the quantized Hall conductance is expressed by the winding number of the fermionic propagator in the momentum space [10]. Other applications of the axial anomaly to the condensed matter physics are studied in the field of the superfluid ^3He in 3D and in charge density waves in

1D conductors [1,11].

The phenomena caused by the axial anomaly are related to the topologically quantized numbers. On the analogy of QHE, it is expected that the axial anomaly also plays important role in other P- and T-violating 2D systems. In this letter, we investigate the axial anomaly in a quasi-1D chiral superfluid model in 2D, which has the spin-triplet $\varepsilon_x p_x + i\varepsilon_y p_y$ -wave symmetry. P and T-violation occur whenever both of ε_x and ε_y are non-zero. We show that the axial anomaly in 1D causes an accumulation of the mass density of the quasiparticle in an inhomogeneous magnetic field. The axial anomaly also causes a chiral current density, which is perpendicular to the gradient of the magnetic field. These effects are related to the winding number of the gap; $\text{sgn}(\varepsilon_x \varepsilon_y)$ [12]. Our discussion would be valid for the superconductors by taking into account the Meissner effect. By varying the parameters ε_x and ε_y , the model could be applicable to Sr_2RuO_4 near the second superconducting transition point [13], and some quasi-1D organic superconductors [14–16] or the fractional quantum Hall (FQH) state at $\nu = 5/2$ Landau level (LL) filling factor [17–19]. We use the 2+1-dimensional Euclidian spacetime and the natural unit ($\hbar = c = 1$) in the present paper.

Let us consider a quasi-1D chiral superfluid model. We assume a linearized fermion spectrum and a spin-triplet chiral p -wave gap near the Fermi surface in the normal state written as,

$$\epsilon_{R,L}(\mathbf{p}) = \pm v_F(p_x \mp p_F), \quad (1)$$

$$\Delta(\mathbf{p}) = i\sigma_3\sigma_2 \frac{\Delta}{|p_F|}(\varepsilon_x p_x + i\varepsilon_y p_y), \quad (2)$$

where v_F and p_F are the Fermi velocity and the Fermi momentum, respectively. $\epsilon_R(\mathbf{p})$ ($\epsilon_L(\mathbf{p})$) is the kinetic energy for the right (left) mover. When $\varepsilon_x \ll 1$ and $\varepsilon_y \sim 1$, the model describes the low energy excitations (the quasiparticle excitations around the tiny gap points) of Sr_2RuO_4 near the second superconducting transition point under the uniaxial pressure in the $x-y$ plane (the basal plane) [13]. For simplicity, we assume a circular Fermi Surface in the normal state (Fig.1(a)). When $\varepsilon_x \sim 1$ and $\varepsilon_y \ll 1$,

the model describes the excitations near $p_x = \pm p_F$ with the chiral p -wave gap, whose kinetic energy in the normal state is $\epsilon(\mathbf{p}) = -2t_x \cos(p_x a) - 2t_y \sin(p_y b) - \epsilon_F$ ($t_y \ll t_x, \epsilon_F$; the Fermi energy). The model in this case is applicable to some quasi-1D organic superconductors or the FQH state at $\nu = 5/2$ LL filling factor (Fig.1(b)). The quasi-1D superconductivity has been observed in organic conductors, such as (TMTSF)₂X [14]. The NMR knight shift study in Ref. [15] is a evidence supporting a spin-triplet pairing state in (TMTSF)₂PF₆. The spin-triplet superconductivity in a quasi-1D system with a nodeless gap is obtained theoretically when an electron-phonon coupling and antiferromagnetic fluctuations are taken into account [16]. Our discussion would be valid for such superconductors if they have the chiral p -wave pairing symmetry. It has been pointed out that the unidirectional charge density wave state, which has the belt-shaped Fermi sea like Fig. 1(b), seems to be the most plausible compressible state at the half-filled Landau levels in the quantum Hall system [17]. Recently, the FQH effect has been observed at $\nu = 5/2$ [18], and the effect could be described by the chiral p -wave pairing state (the Pfaffian state) [19]. Therefore, our model could be a candidate of the $\nu = 5/2$ FQH state.

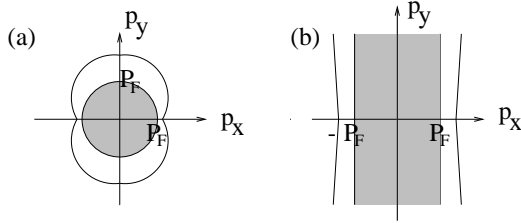


Fig. 1. The Fermi sea in the normal state and the momentum dependence of the gap functions for (a) Sr₂RuO₄ near the second superconducting phase transition point, which corresponds to $\epsilon_x \ll 1$, $\epsilon_y \sim 1$ in our model, and for (b) some quasi-1D organic superconductors with the chiral p -wave pairing symmetry or the FQH state at $\nu = 5/2$ LL filling factor, which corresponds to $\epsilon_x \sim 1$, $\epsilon_y \ll 1$. The shadows show the Fermi sea, and the distance between outer lines and inner lines shows the magnitude of the gap.

The Lagrangian of our model is written as

$$\mathcal{L} = \bar{\Psi}_{\mathbf{p}} \left[\{i\partial_{\tau} + \mu \frac{dB_z}{dy} y \sigma_3\} \otimes \gamma_{\tau} + \frac{\Delta}{|p_F|} \sigma_3 \otimes (\epsilon_x p_x \gamma_x + \epsilon_y p_y \gamma_y) - i v_F (p_x - p_F) \right] \Psi_{\mathbf{p}}. \quad (3)$$

Here, we use the Bogoliubov-Nambu representation with an isospin $\alpha = 1, 2$

$$\Psi(\mathbf{x}) = e^{ip_F \mathbf{x}} \begin{pmatrix} \psi(p_F, \mathbf{x}) \\ i\sigma_2 \psi^*(-p_F, \mathbf{x}) \end{pmatrix},$$

and $\Psi_{\mathbf{p}}$ is its Fourier transform. $\psi(p_F, \mathbf{x})$ and $\psi(-p_F, \mathbf{x})$

are the slowly varying fields for the right mover and the left mover with a real spin index $s = 1, 2$, respectively. The matrices $\gamma_{\tau} = \tau_3, \gamma_x = -\tau_2$ and $\gamma_y = -\tau_1$ are the 2×2 Pauli matrices with isospin indices and σ_i ($i = 1, 2, 3$) is the 2×2 Pauli matrices with spin indices. The symbol $\sigma_i \otimes \gamma_{\tau, x, y}$ shows the direct product. $\bar{\Psi}$ is defined as $\bar{\Psi} = -i\Psi^{\dagger} \gamma_{\tau}$. We assume a magnetic field, which is directed to the z -axis (the c -axis in the crystal) and has a constant gradient in the y -direction, i.e. $B_z(y) = (dB_z/dy)y$, and $(dB_z/dy) = \text{const}$. The magnetic field couples with the Fermion through the Zeeman term $\mu B_z \bar{\Psi} \sigma_3 \otimes \gamma_{\tau} \Psi$, where μ is the magnetic moment of the Fermion. We note that the Lagrangian is similar to that of 2D Dirac QED in a background scalar potential, except for the last term and the existence of σ_3 . The axial anomaly in such a system is discussed in Ref. [7,8].

Let us calculate the expectation value of the mass density

$$\langle \rho_e(\mathbf{x}) \rangle = e \langle \bar{\Psi}(\mathbf{x}) \Psi(\mathbf{x}) \rangle \quad (4)$$

$$= \text{Tr} \left[\frac{e \sigma_3}{\not{d} + \frac{\Delta}{|p_F|} \epsilon_x p_x \hat{1} \otimes \gamma_x - i v_F (p_x - p_F) \sigma_3 \otimes \hat{1}} \right],$$

where e shows a mass of the quasiparticle. It shows an electric charge when we consider a superconductor. A hermitian operator \not{d} in the y -direction is defined as

$$\not{d} = \left\{ i\partial_{\tau} \sigma_3 + \mu \frac{dB_z}{dy} y \right\} \otimes \gamma_{\tau} + \frac{\Delta}{|p_F|} \epsilon_y p_y \hat{1} \otimes \gamma_y. \quad (5)$$

We define γ_5 as $\gamma_5 = i\gamma_{\tau} \gamma_y = -\gamma_x$, and it is anti-commute with γ_{τ} and γ_y , therefore, γ_5 is a hermitian matrix and satisfies $\{\gamma_5, \not{d}\} = 0$. These facts suggest that if an eigenstate u_n of \not{d} with a nonzero eigenvalue ξ_n ($0 < n$) exists (i.e. $\not{d}u_n = \xi_n u_n$), $\gamma_5 u_n$ should be another eigenstate with an eigenvalue $-\xi_n$. If zeromodes of \not{d} exist (i.e. $\not{d}u_0 = 0$ and $\not{d}\gamma_5 u_0 = 0$), they are divided into two groups. One of them is $u_0^{(+)} = (1/2)(1 + \gamma_5)u_0$ with an eigenvalue $\gamma_5 = +1$ and another is $u_0^{(-)} = (1/2)(1 - \gamma_5)u_0$ with an eigenvalue $\gamma_5 = -1$, since $\gamma_5^2 = 1$.

Let us research eigenmodes of \not{d} . The expectation value of \not{d}^2 is

$$(u_n, \not{d}^2 u_n) = |\omega_c| \left(n + \frac{1}{2} \right) + \frac{\omega_c}{2} (u_n, \gamma_5 u_n),$$

$$\omega_c = \mu \frac{dB_z}{dy} \frac{2\Delta}{|p_F|} \epsilon_y, \quad (6)$$

where $u_n = u_n(y - y_c(p_{\tau}, \sigma_3))$ is the eigenfunction of the harmonic oscillator with the frequency ω_c . The oscillator is centered at $y_c(p_{\tau}, \sigma_3) = -(dB_z/dy)^{-1} (p_{\tau}/\mu) \sigma_3$. Eq. (6) indicates that only zeromodes which belong to u_0^{-} (u_0^{+}) exist when $0 < \omega_c$ ($\omega_c < 0$). It suggests the nonconservation of the vacuum expectation value of the axial charge which is defined in the second-quantized formalism as

$$\begin{aligned}\langle Q_5 \rangle &= \langle N_+ - N_- \rangle, \\ N_{\pm} &= \int dp_y \hat{u}_0^{\dagger(\pm)} \hat{u}_0^{(\pm)},\end{aligned}\quad (7)$$

while the classical 1D theory $\mathcal{L}_{1D} = \bar{\Psi} \not{D} \Psi$ has the axial symmetry $\Psi \rightarrow e^{i\alpha\gamma_5} \Psi$, i.e. *the axial anomaly occurs*. Here, \hat{u}_0^{\pm} is a second-quantized fermionic field. The anomaly comes from the spectral asymmetry of zero-modes as same as the discussions in Refs. [7,8,20]. In the free system, the energy spectrum of $u_0^{(\pm)}$ is $p_0 = \pm \frac{\Delta}{|p_F|} \varepsilon_y p_y$ in Minkowski spacetime, and all of the negative energy states are filled while all of the positive energy states are empty and $\langle Q_5 \rangle = 0$. After we turn on the magnetic field adiabatically (for a while, we assume $0 < \omega_c$), the energy spectrum of $u_0^{(+)}$ is lowered and $\langle N_+ \rangle$ decreases (i.e. empty negative energy states arise on the spectrum of $u_0^{(+)}$), on the other hand, the energy spectrum of $u_0^{(-)}$ is lifted and $\langle N_- \rangle$ increases (i.e. filled positive energy states arise on the spectrum of $u_0^{(-)}$), therefore $\langle Q_5 \rangle$ does *not* conserve. Finally $\langle N_+ \rangle = 0$ and only $u_0^{(-)}$ exists. The nonzero eigenvalues of \not{D}^2 is $E_n = \omega_c(n + 1/2)$, since the inner product $(u_n, \gamma_5 u_n)$ vanishes whenever $\not{D} u_n \neq 0$ because of the orthogonal relation between the eigenfunctions of the hermitian operator.

Next, we consider the eigenvalue problem of a 2D operator

$$\mathcal{D} = \not{D} + \frac{\Delta}{|p_F|} \varepsilon_x p_x \hat{1} \otimes \gamma_x = \not{D} - \frac{\Delta}{|p_F|} \varepsilon_x p_x \hat{1} \otimes \gamma_5. \quad (8)$$

Let

$$\varphi_n = (\alpha_n u_n + \beta_n \gamma_5 u_n) e^{ip_x x} \quad (9)$$

stands for an eigenfunction. We use a representation for the n -th level such as (\nearrow)

$$\begin{aligned}\langle \rho_e(\mathbf{x}) \rangle &= \text{Tr} \left[\frac{e\sigma_3}{\mathcal{D} - iv_F(p_x - p_F)\sigma_3 \otimes \hat{1}} \right] = \sum_n \int_{-\infty}^{\infty} \frac{dp_{\tau}}{2\pi} \int \frac{dp_x}{2\pi} \text{tr} \left[\frac{e\sigma_3 |u_n(y - y_0(p_{\tau}, \sigma_3))|^2}{\zeta_n(p_x) - iv_F(p_x - p_F)\sigma_3} \right] \\ &= \frac{-e\mu}{2\pi} \frac{dB_z}{dy} \sum_{n \neq 0} \int \frac{dp_x}{2\pi} \text{tr} \left[\frac{1}{\zeta_n^{(+)}(p_x) - iv_F(p_x - p_F)\sigma_3} + \frac{1}{\zeta_n^{(-)}(p_x) - iv_F(p_x - p_F)\sigma_3} \right] \\ &\quad - \frac{e\mu}{2\pi} \frac{dB_z}{dy} \int \frac{dp_x}{2\pi} \text{tr} \left[\frac{1}{\zeta_0(p_x) - iv_F(p_x - p_F)\sigma_3} \right] = -\text{sgn}(\varepsilon_x \varepsilon_y) e\mu N_{1D}(0) \frac{dB_z}{dy},\end{aligned}\quad (14)$$

where the symbol *tr* means a trace on the real spin, and we use the normal-orthogonal relation $\int dy |u_n|^2 = 1$. $\int \frac{dp_x}{2\pi} = \int_{p_F - \Lambda}^{p_F + \Lambda} \frac{dp_x}{2\pi}$, and Λ is a momentum cutoff. We assume a relation $|\Delta| \ll \Lambda^2/2m \ll \epsilon_F$. $N_{1D}(0) = (2\pi v_F)^{-1}$ is the density of state at the Fermi surface in 1D. All of the $n \neq 0$ parts are canceled out because of the co-existence of the eigenvalues $\zeta_n^{(+)}$ and $\zeta_n^{(-)}$. Only

$$\not{D} = \begin{pmatrix} \xi_n & 0 \\ 0 & -\xi_n \end{pmatrix}, u_n = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (10)$$

where,

$$\xi_n = \begin{cases} \sqrt{|\omega_c|(n + \frac{1}{2})} & (n = 1, 2, \dots), \\ 0 & (n = 0). \end{cases}$$

Therefore, the eigenvalue equation is written as

$$\begin{pmatrix} \xi_n & -\frac{\Delta}{|p_F|} \varepsilon_x p_x \\ -\frac{\Delta}{|p_F|} \varepsilon_x p_x & -\xi_n \end{pmatrix} \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} = \zeta_n \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix}. \quad (11)$$

There are two eigenstates for an oscillator in the $n(\neq 0)$ -th level written as

$$\begin{aligned}\zeta_n^{(\pm)}(p_x) &= \pm \sqrt{\xi_n^2 + \frac{\Delta^2}{p_F^2} \varepsilon_x^2 p_x^2}, \\ \begin{pmatrix} \alpha_n^+ \\ \beta_n^+ \end{pmatrix} &= \frac{1}{C_+} \begin{pmatrix} \zeta_n^{(+)} + \xi_n \\ -\frac{\Delta}{|p_F|} \varepsilon_x p_x \end{pmatrix}, \\ \begin{pmatrix} \alpha_n^- \\ \beta_n^- \end{pmatrix} &= \frac{1}{C_-} \begin{pmatrix} \frac{\Delta}{|p_F|} \varepsilon_x p_x \\ -\zeta_n^{(-)} + \xi_n \end{pmatrix},\end{aligned}\quad (12)$$

where C_{\pm} are normalization constants, but for $n = 0$, there is only one eigenstate

$$\begin{aligned}\zeta_0(p_x) &= \frac{\omega_c}{|\omega_c|} \frac{\Delta}{|p_F|} \varepsilon_x p_x, \\ \begin{pmatrix} \alpha_0 \\ \beta_0 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} 1 \\ -\omega_c/|\omega_c| \end{pmatrix},\end{aligned}\quad (13)$$

because the solution should satisfy $\gamma_5 \varphi_0 = -(\omega_c/|\omega_c|) \varphi_0$. This condition comes from the axial anomaly in the y -direction.

Finally, we show the accumulation of the mass density from Eq. (4), which is derived as

the $n = 0$ part survives because of the axial anomaly in the y -direction.

We can define a chiral transformation in the x -direction such as $\psi(\pm p_F, \mathbf{x}) \rightarrow e^{\pm i\alpha} \psi(\pm p_F, \mathbf{x})$, therefore $\Psi \rightarrow e^{i\alpha} \Psi$, $\bar{\Psi} \rightarrow \bar{\Psi} e^{i\alpha}$. The expectation value of the corresponding current density which is perpendicular to dB_z/dy is

$$\langle j_x^{chi}(\mathbf{x}) \rangle = ev_F \langle \bar{\Psi}(\mathbf{x}) \Psi(\mathbf{x}) \rangle = -\text{sgn}(\varepsilon_x \varepsilon_y) \frac{e\mu}{2\pi} \frac{dB_z}{dy}, \quad (15)$$

and we call it a chiral Hall current density.

These two effects are related to the winding number of the gap Eq. (2) [12],

$$\text{sgn}(\varepsilon_x \varepsilon_y) = \int \frac{d^2 p}{16\pi} \text{tr}[\hat{\mathbf{g}} \cdot (\nabla \hat{\mathbf{g}} \times \nabla \hat{\mathbf{g}})], \quad (16)$$

$$\mathbf{g}(\mathbf{p}) = \begin{pmatrix} \text{Re}[\Delta(\mathbf{p})(-i\sigma_2)] \\ -\text{Im}[\Delta(\mathbf{p})(-i\sigma_2)] \\ (\mathbf{p}^2/2m) - \epsilon_F \end{pmatrix},$$

where $\nabla = \partial/\partial\mathbf{p}$. It suggests that these effects occur even if ε_x and/or ε_y are infinitesimally small, and that these effects come from the P- and T-violation of the gap.

The accumulated mass density and the chiral Hall current density exist in the bulk region of the superfluid. In the superconductors, the Meissner effect occurs and the magnetic field cannot penetrate into the bulk, therefore the accumulated charge density and the chiral Hall current density would exist near the edge of the superconductors [21] and also around the vortex core. As we mentioned before, our discussion could be applicable to Sr_2RuO_4 near the second superconducting phase transition point, some quasi-1D organic superconductors and the FQH state at $\nu = 5/2$ LL filling factor by varying the parameters ε_x and ε_y . Recently, the vortex in chiral superconductors has been discussed [22], and such a vortex has a fractional charge and a fractional angular momentum. Interesting phenomena related to these fractional quantum numbers and the present effects are expected to occur around the vortex core.

The axial anomaly also causes the spin quantum Hall effect (SQHE) in the chiral d -wave ($d_{x^2-y^2} + id_{xy}$ -wave) superconductors [23]. The low energy quasiparticles in a magnetic field with a constant gradient can be mapped onto the massive Dirac Fermion in a constant electric field, and the spin rotation around the z -axis for the quasiparticle corresponds to the $U(1)$ transformation for the Dirac Fermion. Therefore, according to the discussions in Ref. [7,8], we can see that the axial anomaly causes the quantized spin Hall current, which is perpendicular to the gradient of the magnetic field.

SQHE has been pointed out by Volovik and Yakovenko in superfluid ^3He -A film, which is the chiral p -wave superfluid [1,24]. They have described the effect by the Chern-Simons term. It has been clarified the relation between the axial anomaly and the Chern-Simons term in 2D Dirac QED [8]. Therefore, SQHE in ^3He -A could be related to the axial anomaly. According to Ref. [24], SQHE also occurs at the edge or around the vortex core of the superconducting Sr_2RuO_4 [2] by a magnetic field in the basal plane.

The author thanks K. Ishikawa and N. Maeda for useful discussions and encouragement. This work was par-

tially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sports, and Culture, the Grant-in-Aid for Scientific Research on Priority area (Physics of CP violation) (Grant No. 10140201), and the Grant-in-Aid for International Science Research (Joint Research 10044043) from the Ministry of Education, Science, Sports and Culture, Japan.

-
- [1] See, for example, G. E. Volovik, “*Exotic Property of Superfluid ^3He* ”, World Scientific, Singapore (1992).
 - [2] G. M. Luke, *et. al.*, Nature **394**, 558-561 (1998).
 - [3] K. Krishana *et. al.*, Science **277**, 83 (1997); R. B. Laughlin, Phys. Rev. Lett. **80** 5188 (1998).
 - [4] S. L. Adler, Phys. Rev. **177** 2426 (1969); W. Bardeen, *ibid.* **184** 1848 (1969).
 - [5] J. S. Bell and R. Jackiw, Nuovo Cim. **60A** 47 (1969).
 - [6] K. Fujikawa, Phys. Rev. Lett. **42** 1195 (1979); *ibid.* **44** 1733 (1980); M. Atiyah and I. Singer, Ann. Math. **87**, 484 (1968).
 - [7] A. Niemi and G. Semenoff, Phys. Rev. Lett. **51**, 2077 (1983); A. Redlich, *ibid.* **52** 18 (1984).
 - [8] K. Ishikawa, Phys. Rev. Lett. **53** 1615 (1984); Phys. Rev. D **31** 1432 (1985).
 - [9] X. G. Wen, Phys. Rev. Lett. **64** 2206 (1990); N. Maeda, Phys. Lett. B, **376** 142 (1996), and references therein.
 - [10] K. Ishikawa and T. Matsuyama, Z. Phys. C **33**, 41 (1986); Nucl. Phys. **B280**, 523 (1987); N. Imai, *et. al.*, Phys. Rev. B **42**, 10610 (1990).
 - [11] G. E. Volovik, Sov. Phys. JETP **65**, 1193 (1987); M. Stone and F. Gaitan, Annals of Phys. **178**, 89 (1987); B. Sakita and K. Shizuya, Phys. Rev. B, **42** 5586 (1990).
 - [12] G.E. Volovik, Sov. Phys. JETP **67**, 1804 (1988); J. Goryo and K. Ishikawa, Phys. Lett. A **260**, 294 (1999).
 - [13] M. Sigrist, R. Joynt and T. M. Rice, Europhys. Lett., **3** 629 (1987).
 - [14] See, for reviews, T. Ishiguro and K. Yamaji, *Organic superconductors* (Springer, 1990).
 - [15] I. J. Lee, *et. al.*, cond-mat/0001332.
 - [16] M. Kohmoto and M. Sato, cond-mat/0001331.
 - [17] N. Maeda, Phys. Rev. B **61** 4766 (2000).
 - [18] W. Pan *et. al.*, Phys. Rev. Lett. **83** 3530 (1999).
 - [19] G. Moore and N. Read, Nucl. Phys. **B360** 362 (1991).
 - [20] R. Jackiw, The Proceedings of “*Lectures in Les Houches summer school, Session XL*”, 221 (1983).
 - [21] Note that the axial anomaly discussed here is in the y -direction, which is perpendicular to the edge, therefore it is different from that of the chiral edge mode [9].
 - [22] J. Goryo, Phys. Rev. B **61** 4222 (2000).
 - [23] T. Senthil, J. B. Marston and M. P. A. Fisher, cond-mat/9902062.
 - [24] G. E. Volovik and V. M. Yakovenko, J. Phys. Condens. Matter **1** 5263 (1989). See, also N. Read and D. Green, cond-mat/9906453.